Analysis of the Influence of Temperature and Humidity on Rainfall in Sindh Province by Vector Auto regression (VAR)

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The effects of climate change are relatively local, although it is a worldwide problem. Since climate change has already occurred, it has had a wide variety of effects in almost all regions of the country and has also had an impact on many economic sectors. Rainfall, temperature, cloud cover, wind speed, humidity, and heavy sunlight are the main climatic variables. In order to understand the subsequent changes of these climatic variables, the behaviour of these variables should be studied. which also helps to implement significant policies. Investigating the behaviour of the climatic factors in the past, present, and future is a major problem. Primary goal of this research project was to create an adequate vector autoregression (VAR) model that could forecast monthly temperature, humidity, and rainfall at three meteorological stations in Sindh Province, Pakistan. The Kwiatkowski–Phillips-Schmidt–Shin, Phillips–Perron, and Augmented Dickey–Fuller tests have all verified the stationarity of time series variables. Order of the VAR model was determined by applying Schwarz information criteria, Hannan-Quinn information criteria, Akaike information criteria, final prediction error, and likelihood ratio test. Ordinary least squares approach was the method utilized to estimate the parameters of the model. It was determined that the optimal models for this study were VAR (8) and VAR (7). The structural analyses were performed using the forecast error variance decomposition and impulse response function. These structural studies show that, in the future, humidity, temperature, and rainfall will all be endogenous. The research indicates that humidity and temperature both favour rainfall. When temperature and humidity are both high, rainfall is greatest, and when they are both low, it is minimum. This study suggests that the correlation between temperature and relative humidity holds negligible influence over changes in rainfall patterns.

1. Introduction

Pakistan, as a developing country, is probably going to be among the most seriously impacted by changing climate. The Global Climate Risk Index for 2023-24 states that Pakistan ranks fifth among countries in terms of vulnerability to the negative effects of global warming. Pakistan is set to face a rise in highly intense weather phenomena, including the devastating floods recorded in 2022. In recent past years, the monsoon season resulted in extensive and fatal flooding that impacted approximately 15% of the entire population, as scorching heat waves were followed by the most severe rainfall and flooding ever recorded in the country (Zhai et al., 2024).

Over the last two decades, climate change has gained international attention due to its predicted consequences on the environment of states that are vulnerable. Temperatures are steadily rising, and this has an effect on rainfall and the cryosphere in many parts of the world. Based on available data, Pakistan appears to have experienced some of the significant climate fluctuations that were previously documented in northwest India. The main cause of the climatic fluctuations was spatial variations in rainfall patterns, which were connected to variations in general atmospheric circulation in the region (Wang et al., 2024).

Changes in rainfall patterns directly affect the water, agriculture, international trade, and disaster management sectors. The 2020 Task Force on Climate Change in Pakistan research states that Pakistan is vulnerable to several natural calamities, including earthquakes, cyclones, floods, droughts, and severe rains. Extreme weather events have become more common and intense over the past few decades. According to (Khan et al., 2022), approximately 40% of people are highly vulnerable to frequent multiple disasters, including variations in rainfall patterns, storms, floods, and droughts. The impact was analyzed in the economy of china for different patterns to be utilized in direction and context (Zhai et al., 2024).

This study focuses on three specific climatic variables—rainfall, maximum temperature, and humidity, to explore their interrelationships and overall climate conditions. The long-term weather pattern is better understood with the use of time series modelling and forecasts. The fact that climate change is currently a worldwide problem makes this study significant. Initially, we have attempted to ascertain the pattern of this issue in order to solve it. An essential tool for illustrating this pattern is the vector autoregression (VAR) model. This technique can forecast connected variables, evaluate how random disturbances affect the system dynamically, and look into the consequences of shocks to associated time series variables. Climate-related factors have been the subject of several studies.

The trends of temperature, rainfall, relative humidity, and sunlight were analyzed using the ordinary least squares approach (Ahmad & Jabeen, 2023). Some researchers modelled meteorological factors as temperature, relative humidity, and precipitation using stochastic methods (ARIMA model, autoregressive integrated moving average). One crucial area of research is the fluctuation of climate variables. Such analyses are provided by VAR, and many different fields make extensive use of these analyses. (Lobel, 2016) used the VAR model for exchange rate and trade balance. The VAR model was employed by (Wu & Zhu, 2017) to analyse macroeconomic factors related to Pakistan's economic growth. Structural VAR models were employed by different researchers in their causal search. For daily weather data, the vector autoregressive moving average (VARMA) process model used by (Durban & Glasbey, 2001) was the vector second-order autoregressive, first-order moving average process.

This model produced a more realistic simulated series and fit the data better than previous models. Bivariate VAR time series models were utilised by (Fatemi & Narangifard, 2019) to match the daily sea surface temperatures and North Atlantic Oscillation time series obtained from a 50-year simulation of the Third Hadley Centre Coupled Ocean–Atmosphere GCM (HadCM3). (Wang & Niu, 2019) used the VAR model to determine the dew points, soil temperatures, wind speeds, and temperatures in the Los Angeles Long Beach area. Using the VAR model, (Chandio et al, 2020) examined how Pakistani wheat output was affected by climate change and came to the conclusion that there was no discernible adverse effect.

(De Sousa et al., 2021) found that temperature and rainfall time series in Niger State of Nigeria, were bi-directionally causal, using the VAR model to analyze the dynamic link between them. It is therefore essential to look at all the related aspects in order to get a more accurate forecast based on climatic data. The purpose of this research is to develop an appropriate VAR model for better meteorological data forecasting, including humidity, maximum temperature, and rainfall, for the chosen weather station in Pakistan's Sindh region. When the review was analyzed on the basis of african region which worked for the analysis of temperature and restoration (Mansorian & Berrahmouni, 2021).

1.1 Objectives of Study

i) To assess the rainfall, temperature, cloud cover, wind speed, humidity, and heavy sunlight as the main climatic variables.

ii) To understand the subsequent changes of these climatic variables, the behaviour of these variables as studied with the help of implementation of significant policies.

1.2 Research Questions

i) What is impact of Rainfall, temperature, cloud cover, wind speed, humidity, and heavy sunlight on the climatic changes?

ii) How to analyze the subsequent changes of these climatic variables and the behaviour of these variables?

2. Literature Review

Variations in the region's overall atmospheric circulation were linked to changes in rainfall patterns, which accounted for the majority of the climatic oscillations (Rodo, 2003). Variations in precipitation trends have an immediate impact on the water, agricultural, international trade, and disaster management industries. According to study conducted in 2010 by the Task Force on Climate Change in Pakistan, Pakistan is susceptible to a number of natural disasters, such as earthquakes, cyclones, floods, droughts, and heavy rains. Over the past few decades, extreme weather events have increased in frequency and intensity.

According to (Kukal & Irmak, 2018), approximately 40% of people are highly vulnerable to frequent multiple disasters, including variations in rainfall patterns, storms, floods, and droughts. The impact was analyzed in the economy of china for different patterns to be utilized in direction and context (Zhai et al., 2024).

The main focus of this study is on rainfall, maximum temperature, and humidity in order to look into the relationships between these three distinct climatic elements and the overall environment. We can better comprehend the long-term weather trend with the use of forecasts and time series models. Since climate change is currently a global concern, this work is crucial. Initially, we have attempted to identify the pattern of this issue in order to resolve it. The vector autoregression (VAR) model is an essential tool for illustrating this trend.

Predicting linked variables, evaluating the dynamic consequences of random disturbances on the system, and examining the effects of shocks to connected time series variables can all be done with this method. Many studies have looked at various elements related to climate. The trends in temperature, precipitation, relative humidity, and sunshine were evaluated using the ordinary least squares approach (Ahmad & Jabeen, 2023)

Using stochastic techniques, one author simulated meteorological variables such as temperature, relative humidity, and precipitation (ARIMA model, autoregressive integrated moving average). Climate variable fluctuation is an important research area. These analyses are offered by VAR, and they are heavily utilized by numerous fields. The VAR model was applied to democracy and trade balance (Khan & Hossain, 2010). (Altaf et al., 2012) used the VAR model to analyze macroeconomic variables associated with Pakistan's economic growth.

Several researchers used structural VAR models in their causal investigation. The vector second-order autoregressive, first-order moving average process was the vector autoregressive moving average (VARMA) process model employed by (Durban and Glasbey, 2001) for daily meteorological data. Compared to earlier models, this one fit the data better and generated a simulated series that was more realistic. In order to match the daily sea surface temperatures and North Atlantic Oscillation time series derived from a 50-year simulation of the Third Hadley Center Coupled Ocean–Atmosphere GCM (HadCM3), Mosedale et al. (2016) employed bivariate VAR time series models.

The dew points, soil temperatures, wind speeds, and temperatures in the Los Angeles Long Beach region were all measured by Wang and Niu (2019) using the VAR model. (Chandio et al., 2020) looked at how climate change influenced Pakistan's wheat output using the VAR model and found no appreciable negative effects. (Adenomon et al., 2013) used the VAR model to examine the dynamic relationship between temperature and rainfall time series in Niger State, Nigeria, and discovered that they were bi-directionally causative. Therefore, in order to obtain a more accurate forecast based on meteorological data, it is imperative to consider all relevant factors.

The goal of this study is to create a suitable VAR model for the selected weather station in the Sindh region of Pakistan in order to improve the forecasting of meteorological data, such as humidity, maximum temperature, and rainfall. When the review was examined using the African region as a basis, temperature and restoration analyses were conducted (Mansorian & Berrahmouni, 2021).

3. Research Methodology

According to Lutkepohl (2005), a time series is essentially a collection of measurement data that was taken in a certain chronological order. In the current study, the Vector Autoregression (VAR) approach is utilized based on the characteristics of each time series with various types of data (rainfall, maximum temperature, and humidity). Multivariate time series is modelled using vector autoregressive models. As per the suggested structure, each variable is a linear function of both its own past lags and the past lags of the other variables. The Box-Jenkins approach, which was presented in 1976 by Gwilym Jenkins and George Box, is sometimes referred to as the Autoregressive (AR) method and is essentially combined into the VAR (Lutkepohl, 2005).
For example, the AR time series is given as:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots \beta_{p-1} y_{t-p-1} + \beta_p y_{t-p} + \varepsilon_t$$

y_t in above equation is the current value, p denotes lag in autoregressive process, y_{t-1} to y_{t-p} is the measurement values from t-1 to t-p; intercept value is denoted by β_0 whereas regression coefficient from t-1 to t-p represented by β_1 to β_p, ε_t is an error term, often known as white noise, which is supposed to have a normal distribution. Independent of y_t and constant variance of σ^2 or equal to 0 (Cowpertwait and Andrew, 2009. Im et al., 2003).

4. Data

The regional meteorological stations in the province of Sindh were selected for the raw data for this study. The monthly data on rainfall (millimeters), maximum temperature (centigrades), and percentage of humidity from these three metrological stations was gathered. In this study, data from the official Sindh Bureau of Statistics website (https://sbos.sindh.gov.pk/development-statistics-of-Sindh) was collected for the metrological stations of Karachi, Hyderabad, and Sukkur between January 2011 and December 2021.

4.1 Test of Stationary

In order to employ the VAR model in this study, the stationarity condition requirements had to be applied. According to Lutkepohl (2005), a stationary condition is one in which the covariance is not time dependent and model has a constant mean and variance.

4.2 Augmented Dickey–Fuller Test

To verify the stationary of data, the unit root test was performed using the Augmented Dickey-Fuller (ADF) approach (Arltová & Fedorová, 2016). The following mathematical formula is used in the ADF test:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t \quad (1)$$

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{m} \delta_i \Delta y_{t-i} + \varepsilon_t \quad (2)$$

In above both equations Δy_t is the difference at the t time of given time series and its value in 1 previous measurement period, α is a constant, β the coefficient on a time trend, γy_{t-1} is a stationary error-correction term. The unit root test is performed with the null hypothesis $\gamma = 0$and the alternative hypothesis$\gamma < 0$. We are primarily concerned with negative values in our test statistic τ. If statistic test is smaller (more negative) than the essential value, the null hypothesis of $\gamma = 0$ is rejected and no unit root exists. (Nugroho et al., 2014). According to Wei (2019), the error component ε_t in the augmented model is not auto-correlated and is instead classified as white noise.

4.3 Phillips–Perron Test

For the purpose of handling serial correlation without using the augmented term of the ADF equation (Eq. 2), Phillips and Perron (1988) present a nonparametric statistical technique. For the Phillips–Perron (PP) test, firstly γ is estimated from the non-augmented Dickey–Fuller (1979) (Equation 3) as:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \varepsilon_t \quad (3)$$

The statistic equation for PP test is presented as follows:

$$t_{\delta,PP} = t_\delta \sqrt{\frac{\gamma_0}{f_0}} - \frac{N(f_0 - \gamma_0)(SE(\hat{\delta}))}{2\sqrt{f_0}\,s} \quad (4)$$

$\hat{\delta}$ the estimated value of δ, t-ratio of δ is denoted by t_δ, $SE(\hat{\delta})$ represents coefficient of standard error, and test regression's standard error is denoted by s. Furthermore, γ_0 provides a consistent estimate of error variance in the non-augmented Dickey-Fuller equation (Eq. 3) and computed by $(N - K)s^2/N$, symbol K show how many regressors are there. The remaining term, f_o, is an estimator of the residual spectrum at frequency zero.

4.4 Kwiatkowski–Phillips–Schmidt–Shin Test

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Unlike other tests, the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) (1992) test is based on the assumption that the series is stationary under the null hypothesis. The KPSS statistic is based on the residuals from the ordinary least square (OLS) regression of $y_t$on the exogenous variables of lag y_t. For one exogenous variable using one lagged value ofy_t, the regression model can be written as follows:

$$y_t = \gamma y_{t-1} + u_t \qquad (5)$$

We define the LM statistic as follows:

$$LM = \sum \frac{S(t)^2}{N^2 f_0} \qquad (6)$$

In above equation, estimator of the residual spectrum at frequency zero is denoted by f_o and residuals $\hat{u}_r$ serve as the basis for the cumulative residual function $S(t) = \hat{u}_r$

4.5 Constructing an Order p Model

There are two types of variables in a simultaneous equation model; endogenous and exogenous. In a VAR model, every variable is an endogenous variable (Sims, 1980). Granger causality detection procedure, first introduced by (Granger, 1969) and later popularized by (Sims, 1972), and it may be used to assess the endogeneity of the variables. The variables that possess endogenous properties are ultimately chosen for the VAR examination.

If rainfall, maximum temperature and humidity is denoted by R_t, T_t and H_t for $t = 1, 2, \dots, N$, respectively then VAR(p) model, which has three variables of arbitrary order p, can be expressed as follows:

$$R_t = c_1 + a_{11}^1 R_{t-1} + \cdots + a_{1p}^1 R_{t-p} + a_{11}^2 T_{t-1} + \cdots + a_{1p}^2 T_{t-p} + a_{11}^3 H_{t-1} + \cdots + a_{1p}^3 H_{t-p} + \varepsilon_{1t} \qquad (7)$$

$$T_t = c_2 + a_{21}^1 R_{t-1} + \cdots + a_{2p}^1 R_{t-p} + a_{21}^2 T_{t-1} + \cdots + a_{2p}^2 T_{t-p} + a_{21}^3 H_{t-1} + \cdots + a_{2p}^3 H_{t-p} + \varepsilon_{2t} \quad (8)$$

$$H_t = c_3 + a_{31}^1 R_{t-1} + \cdots + a_{1p}^1 R_{t-p} + a_{11}^2 T_{t-1} + \cdots + a_{1p}^2 T_{t-p} + a_{11}^3 H_{t-1} + \cdots + a_{1p}^3 H_{t-p} + \varepsilon_{3t} \quad (9)$$

In matrix form the above system may be written as:

$$\begin{bmatrix} R_t \\ T_t \\ H_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{11}^2 & a_{11}^3 \\ a_{21}^1 & a_{21}^2 & a_{21}^3 \\ a_{31}^1 & a_{31}^2 & a_{31}^3 \end{bmatrix} \begin{bmatrix} R_{t-1} \\ T_{t-1} \\ H_{t-1} \end{bmatrix} + \cdots + \begin{bmatrix} a_{1p}^1 & a_{1p}^2 & a_{1p}^3 \\ a_{2p}^1 & a_{2p}^2 & a_{2p}^3 \\ a_{3p}^1 & a_{3p}^2 & a_{3p}^3 \end{bmatrix} \begin{bmatrix} R_{t-p} \\ T_{t-p} \\ H_{t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \quad (10)$$

The reduced form of VAR process is as follows:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + \varepsilon_t, t = 1,2 \dots N \qquad (11)$$

In equation 11, y_t is 3×1 column matrix, c is a 3×1 column matrix of constants, A_i is a 3×3 matrix and ε_t is a 3×1 column matrix of error terms and assumed $\varepsilon_t \sim NID(0, \hat{\sigma})$.

4.6 Determining the Value of Order p

For the determination of proper lag length in VAR model for the N sample size, we have various methods which are defined by following equations:

$$AIC(p) = \log|\hat{\sigma}(p)| + \frac{2m(p^2+1)}{N} \quad (12)$$

$$SC(p) = \log|\hat{\sigma}(p)| + \log(N)\frac{m(p^2+1)}{N} \quad (13)$$

$$HQ(p) = \log|\hat{\sigma}(p)| + 2\log(\log(N))\frac{m(p^2+1)}{N} \quad (14)$$

$$FPE(p) = \left(\frac{N+mp+1}{N-mp-1}\right)^m |\hat{\sigma}(p)| \quad (15)$$

with $\hat{\sigma}(p) = N^{-1}\sum_{t=1}^{N}\hat{\varepsilon}_t\hat{\varepsilon}_t'$ and $m(p^2+1)$.

4.7 Diagnostic Checking and Forecasting

To confirm stationary, the unit root test is employed whereas normal Q-Q plot is used to verify residual normality and the Durbin-Watson d test (1951) is applied to verify autocorrelation in the chosen VAR(p) model. Forecasting is also affected by the lag values of other endogenous variables. Reduced versions of the VAR models are useful for forecasting because they show the conditional mean of a stochastic process. Forecast values of $\hat{y}_{t+2}, \hat{y}_{t+3} \ldots \hat{y}_{t+h}$ can be determined by the equation:

$$:\hat{y}_{t+1} = \hat{A}_1 y_t + \hat{A}_2 y_{t-1} + \cdots + \hat{A}_p y_{t-p+1} \quad (16)$$

4.8 Impulse Response Function

The impulse response function (IRF) is an additional structural analysis component that shows how the current and future values of each variable change as the current value of VAR errors rises by one unit. The primary assumption for the impulse response function is that all other errors are equal to zero and that the error returns to zero in subsequent periods.

$$y_t = \mu + \varepsilon_t + \psi_1\varepsilon_{t-1} + \psi_2\varepsilon_{t-2} + \psi_3\varepsilon_{t-3} + \cdots \quad (17)$$

In matrix ψ_s is defined as:

$$\psi_s = \frac{\partial y_{t+s}}{\partial \varepsilon_t'}$$

The element of ψ_s in ith row and jth column depicts the effects of a rise of one unit in the jth variable's innovation at date t (ε_{jt}) for the value of the ith variable at time $t + s(y_{i,t+s})$ holding every other invention across all dates and constants.

4.9 Quantifying Forecast Accuracy

To calculate the forecast accuracy error, a variety of criteria and other less common formulae are employed such as:

Mean Absolute Error (MAE): MAE is a metric that measures the average absolute difference between forecasted values and true values. MAE is generally defined as the following equation:

$$MAE = \frac{1}{n}\sum_{t=1}^{N}|F_t - Y_t|$$

Mean Absolute Percentage Error (MAPE): The mean absolute percentage error expresses the average amount of error generated by a model, or the average deviation from expectations. It can be calculated by:

$$MAPE = \frac{1}{n}\sum_{t=1}^{N}\left|\frac{Y_t - F_t}{Y_t}\right|$$

Root Mean Square Error (RMSE): The root mean square error, also known as the root mean square deviation, is one of the metrics most frequently employed to assess the accuracy of forecasts. Using Euclidean distance, it illustrates the deviation between predicted and measured true values. It is defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{N}(Y_t - F_t)^2}$$

In all measures, F_t indicates forecast value whereas Y_t denotes the actual data and N shows length of observation in time series.

4.10 Results and Discussion

4.11 Descriptive Statistical Analysis

With the help of EViews software, descriptive statistical analysis is performed for the mean, maximum, standard deviation, skewness and kurtosis of the rainfall, maximum temperature, and humidity for each of the selected stations. Study suggests, Karachi recorded more rainfall than other cities on average during the chosen time, with a maximum of 366.80 mm. The distribution of rainfall is positively skewed at every station and the rainfall distribution curve is leptokurtic.

Throughout this research highest temperature was recorded 44.6 0C, as indicated by the table 1, which also gives the average maximum temperature recorded at Sukkur station. For all stations, the temperature distribution is negatively skewed, and the Platykurtic curve is shown by the kurtosis. Sukkur and Hyderabad stations reported 89% of the average humidity, which was determined to be greater than in other cities. The humidity distribution for each station also has a negative skew and kurtosis and each station's kurtosis indicates a Platykurtic curve.

4.12 Tests for Stationary

Table No 1: Descriptive Statistical Analysis for the Rainfall, Maximum Temperature and Humidity

	Rainfall			**Maximum Temperature**			**Humidity**		
	Karachi	Hyderabad	Sukkur	Karachi	Hyderabad	Sukkur	Karachi	Hyderabad	Sukkur
Mean	16.57	10.18	9.98	32.81	34.10	34.63	71.95	72.49	70.19
Maximum	366.8	195.3	210.0	38.7	43.00	44.6	86.0	89.0	89.0
Std. Dev.	45.70	28.05	25.35	3.288	5.59	7.43	8.77	7.46	10.60
Skewness	4.88	4.12	4.89	-0.69	-0.59	-0.45	-0.72	-0.24	-0.68
Kurtosis	32.12	22.07	33.56	2.56	2.14	1.84	2.52	2.69	2.89

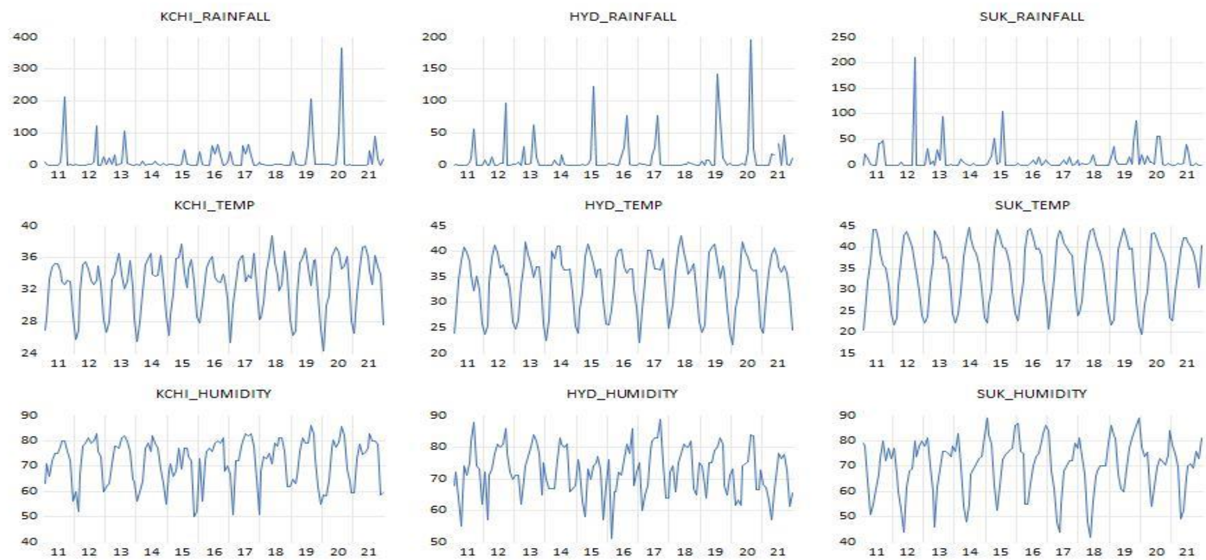
The stationary of the rainfall, maximum temperature, and humidity series has been assessed through the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS), Phillips-Perron test (PP) and Augmented Dickey-Fuller test (ADF), all tests were applied on the first lag. The findings of all three tests show that the humidity, maximum temperature, and rainfall series are stationary, as shown in table 2.The graphical representation of all the series also confirms the stationarity of the data in figure 1.

Table No 2: Unit Root Test for Climatic Variables

	Rainfall		
	KPSS	PP	ADF
Karachi	0.138138	-9.272899	-5.749427
Hyderabad	0.188462	-9.649622	-7.903023
Sukkur	0.110781	-10.95553	-8.161090
	Temperature		
	KPSS	PP	ADF
Karachi	0.131049	-5.844337	-8.813485
Hyderabad	0.021440	-5.160379	-8.771375
Sukkur	0.058361	-4.634678	-9.085833
	Humidity		
	KPSS	PP	ADF

Karachi	0.035643	-5.622820	-5.863417
Hyderabad	0.080000	-6.810709	-6.170325
Sukkur	0.068066	-7.073070	-5.120220

Figure No 1: The Stationary of the Rainfall, Maximum Temperature, and Humidity Series



4.13 VAR Model of Order p Selection Criteria

There are two steps involved in specifying a VAR model: Analyzing the k variables that should be part of the model and choosing the lag order, which is an important decision for the best possible VAR models. The following procedures were taken into consideration for the lag selection criteria in our study: (1) Likelihood ratio test statistics (LRT), (2) Final prediction error (FPE), (3) Akaike information criteria (AIC), (4) Schwarz information criteria (SC), and (5) Hannan-Quinn information criteria (HQ).Table 3 (a) shows results of appropriate lag order at Karachi weather stations. Lag order for VAR should be 8, according to (1) LRT, (2) FPE, and (3) AIC; nevertheless, lag 3 is suggested by (4) SC and lag 4 by (5) HQ.

Similarly, Table 3.3 (b) shows results of appropriate selection of lag order at Hyderabad weather station. According to (1) LRT, (2) FPE, (3) AIC, and (5) HQ, the appropriate lag order for a VAR is 8 but (4) SC suggest lag 3. Meanwhile, Table 3.3 (c) indicates the selection of lag order and results shows appropriate lag order is 7 as suggested by (1) LRT, (2) FPE and (3) AIC whereas (4) SC and (5) HQ suggest lag order 3. AIC has been found to be more accurate with monthly data when used in conjunction with VAR models (Ivanov and Kilian, 2001). Therefore, we have selected VAR (8) for Karachi and Hyderabad weather stations and VAR (7) for Sukkur weather station.

Table No 3: (a) Selection of VAR Model Order (p) for Karachi Station

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-1375.644	NA	911246.2	22.23620	22.30443	22.26392
1	-1319.424	108.8137	425497.7	21.47458	21.74751	21.58545
2	-1278.617	77.00724	254813.7	20.96156	21.43919	21.15558
3	-1251.679	49.52999	190931.2	20.67225	21.35457*	20.94942
4	-1237.195	25.93195	174988.4	20.58379	21.47081	20.94412*
5	-1228.604	14.96506	176502.7	20.59038	21.68210	21.03387
6	-1216.528	20.45128	168457.1	20.54077	21.83719	21.06741
7	-1203.523	21.39438	158567.6	20.47618	21.97730	21.08597
8	-1188.704	23.66348*	145145.1*	20.38232*	22.08813	21.07526

Table No 3: (b) Selection of VAR Model Order (p) for Hyderabad Station

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-1315.713	NA	1237342.	22.54211	22.61293	22.57086
1	-1246.560	133.5788	442540.9	21.51384	21.79714	21.62886
2	-1198.799	89.80653	228218.9	20.85126	21.34704	21.05254
3	-1167.003	58.15742	154694.0	20.46158	21.16983*	20.74912
4	-1153.211	24.51865	142743.4	20.37967	21.30040	20.75347
5	-1145.145	13.92609	145390.3	20.39564	21.52883	20.85570
6	-1131.397	23.03024	134531.2	20.31448	21.66015	20.86081
7	-1105.525	42.01409	101318.7	20.02607	21.58422	20.65866
8	-1090.537	23.57045*	92055.79*	19.92372*	21.69434	20.64257*

Table No 3: (c) Selection of VAR Model Order (p) for Sukkur Station

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-1424.846	NA	2015044.	23.02978	23.09801	23.05750
1	-1295.156	251.0145	287676.3	21.08316	21.35609	21.19403
2	-1246.235	92.31707	151147.4	20.43928	20.91691	20.63331
3	-1223.946	40.98321	122071.6	20.22494	20.90727*	20.50212*
4	-1215.792	14.59918	123904.6	20.23858	21.12560	20.59891
5	-1209.388	11.15587	129463.4	20.28044	21.37217	20.72393
6	-1186.976	37.95516	104589.1	20.06413	21.36055	20.59076
7	-1174.032	21.29545*	98544.62*	20.00051*	21.50163	20.61030
8	-1166.773	11.59075	101901.7	20.02859	21.73441	20.72154

4.14 Parameter Estimation with OLS Method

Ordinary Least Square approach is used to compute the parameters of the best selected VAR model and the outcomes are displayed in Tables 4(a, b, c). The predictor variables and estimated parameters of Eqs, 7, 8, and 9 are corresponding to the first, second, third, and fourth columns in these tables (excluding the first and last rows). The top row displays the response variables, while the last row displays the coefficient of determination of R^2 for Equations 7, 8, and 9 respectively, where the asterisks denote the significance coefficient of the calculated model.

The R2 of the VAR (8) models at Karachi station for rainfall, maximum temperature, and humidity are, respectively, 0.3109, 0.8537, and 0.7633; the means also fit the data. At Hyderabad station, the R^2values of VAR (8) models are 0.378, 0.919, and 0.654 concerning rainfall, maximum temperature, and humidity, respectively. Meanwhile, at Sukkur station, the R^2 values of VAR (7) models are 0.215, 0.929, and 0.809 concerning rainfall, maximum temperature, and humidity, respectively. Figure 2 illustrates the actual and fitted values of the VAR(p) model in place of the maximum temperature, humidity, and rainfall.

Table No 4: (a) The Estimated Coefficients Using Ordinary Least Squares (OLS) for the VAR (8) Model at Karachi Station

Variable	R_t	T_t	H_t
Constant	$C_1 = 172.102$	$C_2 = 36.731*$	$C_3 = 128.703*$
R_{t-1}	$a_{11}^1 = 0.076$	$a_{21}^1 = -0.002$	$a_{31}^1 = 0.002$
R_{t-2}	$a_{12}^1 = -0.084$	$a_{22}^1 = 0.008*$	$a_{32}^1 = 0.004$
R_{t-3}	$a_{13}^1 = -0.009$	$a_{23}^1 = -0.008*$	$a_{33}^1 = -0.018$
R_{t-4}	$a_{14}^1 = -0.066$	$a_{24}^1 = -0.001$	$a_{34}^1 = 0.013$
R_{t-5}	$a_{15}^1 = 0.024$	$a_{25}^1 = -0.005$	$a_{35}^1 = -0.027*$
R_{t-6}	$a_{16}^1 = 0.070$	$a_{26}^1 = 0.010*$	$a_{36}^1 = 0.026*$

R_{t-7}	$a^1_{17} = 0.054$	$a^1_{27} = -0.001$	$a^1_{37} = 0.014$
R_{t-8}	$a^1_{18} = -0.018$	$a^1_{28} = 0.002$	$a^1_{38} = 0.018$
T_{t-1}	$a^2_{11} = 0.080$	$a^2_{21} = 0.500*$	$a^2_{31} = 0.842*$
T_{t-2}	$a^2_{12} = 2.798$	$a^2_{22} = -0.231*$	$a^2_{32} = -0.966*$
T_{t-3}	$a^2_{13} = 0.482$	$a^2_{23} = -0.222*$	$a^2_{33} = 1.057*$
T_{t-4}	$a^2_{14} = 0.021$	$a^2_{24} = 0.074$	$a^2_{34} = -0.198$
T_{t-5}	$a^2_{15} = -1.546$	$a^2_{25} = 0.047$	$a^2_{35} = -0.552$
T_{t-6}	$a^2_{16} = 3.496$	$a^2_{26} = -0.0003$	$a^2_{36} = -0.393$
T_{t-7}	$a^2_{17} = -8.537*$	$a^2_{27} = 0.104$	$a^2_{37} = -0.003$
T_{t-8}	$a^2_{18} = -0.970$	$a^2_{28} = -0.175$	$a^2_{38} = -1.398$
H_{t-1}	$a^3_{11} = -0.758$	$a^3_{21} = 0.106*$	$a^2_{31} = 0.247*$
H_{t-2}	$a^3_{12} = -0.665$	$a^3_{22} = 0.004$	$a^2_{32} = -0.192$
H_{t-3}	$a^3_{13} = 0.829$	$a^3_{23} = -0.024$	$a^2_{33} = 0.085$
H_{t-4}	$a^3_{14} = 0.052$	$a^3_{24} = -0.061*$	$a^2_{34} = -0.231*$
H_{t-5}	$a^3_{15} = -0.180$	$a^3_{25} = -0.030$	$a^2_{35} = 0.002$
H_{t-6}	$a^3_{16} = -0.610$	$a^3_{26} = -0.069*$	$a^2_{36} = -0.114$
H_{t-7}	$a^3_{17} = 722$	$a^3_{27} =-0.022$	$a^2_{37} =0.074$
H_{t-8}	$a^3_{18} = 0.013$	$a^3_{28} = -0.004$	$a^2_{38} = 0.067$
	$R^2 = 0.3109$	$R^2 = 0.8537$	$R^2 = 0.7633$

Table No 4:(b) The Estimated Coefficients using Ordinary Least Squares (OLS) for the VAR (8) Model at Hyderabad Station

Variable	R_t	T_t	H_t
Constant	$C_1 = 79.769$	$C_2 = 58.299 *$	$C_3 = 31.202$
R_{t-1}	$a^1_{11} = -0.026$	$a^1_{21} = -0.008$	$a^1_{31} = -0.008$
R_{t-2}	$a^1_{12} = -0.140$	$a^1_{22} = 0.010$	$a^1_{32} = -0.004$
R_{t-3}	$a^1_{13} = -0.020$	$a^1_{23} = -0.013$	$a^1_{33} = -0.027$
R_{t-4}	$a^1_{14} = 0.055$	$a^1_{24} = -0.013$	$a^1_{34} = -0.002$
R_{t-5}	$a^1_{15} = -0.016$	$a^1_{25} = -0.013$	$a^1_{35} = -0.004$
R_{t-6}	$a^1_{16} = 0.041$	$a^1_{26} = 0.012$	$a^1_{36} = 0.026$
R_{t-7}	$a^1_{17} = 0.039$	$a^1_{27} = 0.005$	$a^1_{37} = -0.037$

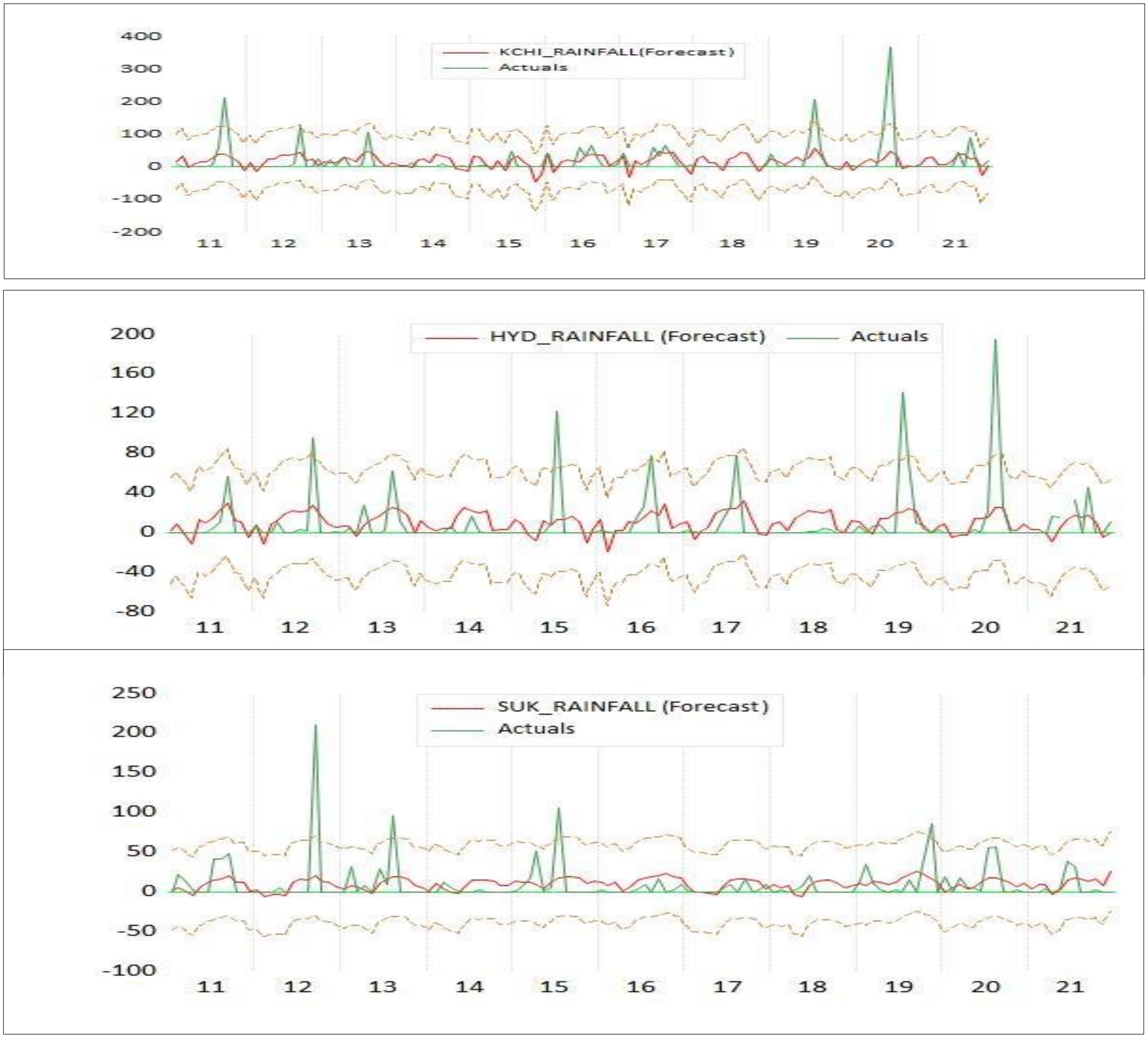
R_{t-8}	$a_{18}^{1} = -0.019$	$a_{28}^{1} = -0.008$	$a_{38}^{1} = -0.027$
T_{t-1}	$a_{11}^{2} = 0.406$	$a_{21}^{2} = 0.488*$	$a_{31}^{2} = 0.319$
T_{t-2}	$a_{12}^{2} = -0.568$	$a_{22}^{2} = -0.064$	$a_{32}^{2} = -0.193$
T_{t-3}	$a_{13}^{2} = 2.094$	$a_{23}^{2} = -0.356$	$a_{33}^{2} = 0.856*$
T_{t-4}	$a_{14}^{2} = 0.319$	$a_{24}^{2} = -0.346*$	$a_{34}^{2} = -0.288$
T_{t-5}	$a_{15}^{2} = -1.974$	$a_{25}^{2} = 0.070$	$a_{35}^{2} = -0.086$
T_{t-6}	$a_{16}^{2} = 2.269$	$a_{26}^{2} = -0.047$	$a_{36}^{2} = 0.013$
T_{t-7}	$a_{17}^{2} = -3.589*$	$a_{27}^{2} = -0.189$	$a_{37}^{2} = -0.297$
T_{t-8}	$a_{18}^{2} = -0.436$	$a_{28}^{2} = -0.374*$	$a_{38}^{2} = -0.112$
H_{t-1}	$a_{11}^{3} = -0.677$	$a_{21}^{3} = 0.039$	$a_{31}^{2} = 0.097$
H_{t-2}	$a_{12}^{3} = 0.062$	$a_{22}^{3} = 0.016$	$a_{32}^{2} = 0.016$
H_{t-3}	$a_{13}^{3} = -0.083$	$a_{23}^{3} = -0.011$	$a_{33}^{2} = 0.063$
H_{t-4}	$a_{14}^{3} = -0.139$	$a_{24}^{3} = -0.022$	$a_{34}^{2} = 0.052$
H_{t-5}	$a_{15}^{3} = 0.036$	$a_{25}^{3} = 0.071*$	$a_{35}^{2} = -0.058$
H_{t-6}	$a_{16}^{3} = -0.096$	$a_{26}^{3} = -0.057$	$a_{36}^{2} = -0.067$
H_{t-7}	$a_{17}^{3} = 0.769$	$a_{27}^{3} = -0.038$	$a_{37}^{2} = 0.155$
H_{t-8}	$a_{18}^{3} = -0.109$	$a_{28}^{3} = 0.056$	$a_{38}^{2} = 0.222*$
	$R^2 = 0.378$	$R^2 = 0.919$	$R^2 = 0.654$

Table No 4: (c) The Estimated Coefficients Using Ordinary Least Squares (OLS) for the VAR (8) Model at Sukkur Station

Variable	R_t	T_t	H_t
Constant	$C_1 = 177.401$	$C_2 = 31.631^*$	$C_3 = 13.136$
R_{t-1}	$a_{11}^1 = -0.135$	$a_{21}^1 = 0.008$	$a_{31}^1 = -0.024$
R_{t-2}	$a_{12}^1 = -0.094$	$a_{22}^1 = -0.003$	$a_{32}^1 = -0.014$
R_{t-3}	$a_{13}^1 = -0.099$	$a_{23}^1 = -0.014$	$a_{33}^1 = -0.011$
R_{t-4}	$a_{14}^1 = 0.081$	$a_{24}^1 = -0.018^*$	$a_{34}^1 = 0.019$
R_{t-5}	$a_{15}^1 = 0.023$	$a_{25}^1 = -0.003$	$a_{35}^1 = 0.004$
R_{t-6}	$a_{16}^1 = -0.056$	$a_{26}^1 = 0.004$	$a_{36}^1 = 0.004$
R_{t-7}	$a_{17}^1 = -0.056$	$a_{27}^1 = 0.004$	$a_{37}^1 = 0.003$
T_{t-1}	$a_{11}^2 = -2.775$	$a_{21}^2 = 0.698^*$	$a_{31}^2 = -0.080$
T_{t-2}	$a_{12}^2 = 1.016$	$a_{22}^2 = 0.125$	$a_{32}^2 = -0.405$
T_{t-3}	$a_{13}^2 = 2.480$	$a_{23}^2 = -0.604^*$	$a_{33}^2 = 1.022^*$
T_{t-4}	$a_{14}^2 = -3.902^*$	$a_{24}^2 = 0.013$	$a_{34}^2 = 0.205$
T_{t-5}	$a_{15}^2 = 1.328$	$a_{25}^2 = 0.089$	$a_{35}^2 = 0.277$
T_{t-6}	$a_{16}^2 = 0.170$	$a_{26}^2 = -0.0003$	$a_{36}^2 = -0.654$
T_{t-7}	$a_{17}^2 = -4.191^*$	$a_{27}^2 = -0.262^*$	$a_{37}^2 = 0.121$
H_{t-1}	$a_{11}^3 = 0.188$	$a_{21}^3 = -0.125^*$	$a_{31}^2 = 0.594$
H_{t-2}	$a_{12}^3 = 0.039$	$a_{22}^3 = 0.146$	$a_{32}^2 = -0.168$
H_{t-3}	$a_{13}^3 = 1.227$	$a_{23}^3 = -0.063$	$a_{33}^2 = 0.202$
H_{t-4}	$a_{14}^3 = -0.206$	$a_{24}^3 = 0.019$	$a_{34}^2 = -0.026$
H_{t-5}	$a_{15}^3 = -0.099$	$a_{25}^3 = -0.098$	$a_{35}^2 = 0.194$
H_{t-6}	$a_{16}^3 = 0.032$	$a_{26}^3 = 0.108^*$	$a_{36}^2 = -0.062$
H_{t-7}	$a_{17}^3 = -0.605$	$a_{27}^3 = 0.032$	$a_{37}^2 = -0.159$

	$R^2 = 0.215$	$R^2 = 0.929$	$R^2 = 0.809$

Figure No 2: Actual and Fitted Values of Rainfall



4.15 Diagnostic Checking

The augmented Dickey-Fuller test (ADF) investigates the null hypothesis that a unit root exists, Durbin-Watson test to determine autocorrelation, and Q-Q plot is used to test normality are the tests that support the diagnostic evaluation of residuals from the chosen VAR (p) model. All residual series exhibit good stationarity, as shown by the results shown in table 5. In particular, an almost normal distribution is revealed by the Q–Q plot for the residual series related to humidity, maximum temperature, and rainfall seen in Figure 3 (a, b, and c). Interestingly, resid01 in each Q-Q graph represents the rainfall graph whereas resid02 and resid03 reflect the temperature and humidity graphs, respectively.

Table No 5: Augmented Dickey–Fuller (ADF) and Durbin–Watson (DW) Residuals Series Test of the VAR (p) Model

		Karachi	
Residuals	ADF-value	5% (C.I.)	DW-value
Rainfall	-11.73355	-2.885051(S)	1.9594*
Max. Temp	-11.59397	-2.885051(S)	2.0032*
Humidity	-10.29094	-2.885051(S)	2.0108*
		Hyderabad	
Rainfall	-10.68839	-2.886509(S)	1.9834*
Max. Temp	-12.16901	-2.886509(S)	1.9996*
Humidity	-11.16282	-2.886509(S)	1.9632*
		Sukkur	
Rainfall	-11.31937	-2.884856(S)	1.9839*
Max. Temp	-9.170096	-2.884856(S)	1.6803*
Humidity	-11.34110	-2.884856(S)	1.9819*

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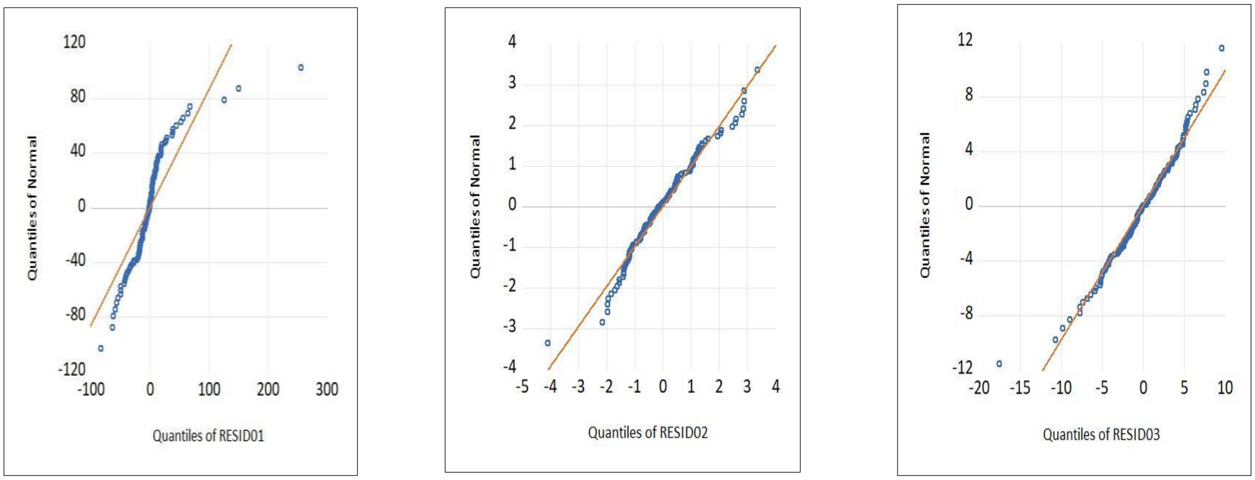


Figure No 3: (a) Q-Q Plot for Karachi Station

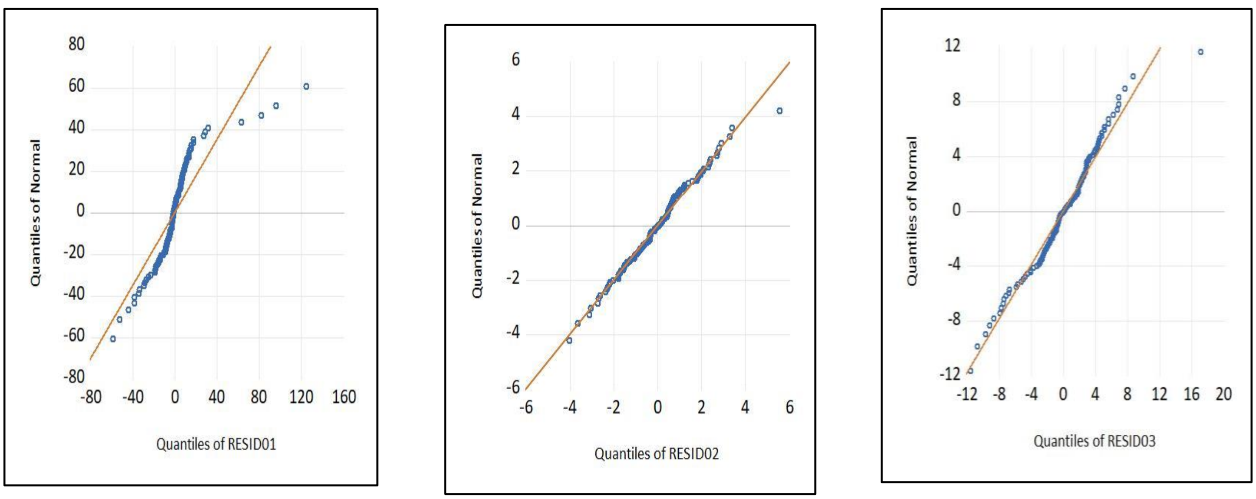


Figure No 3: (b) Q-Q Plot for Hyderabad Station

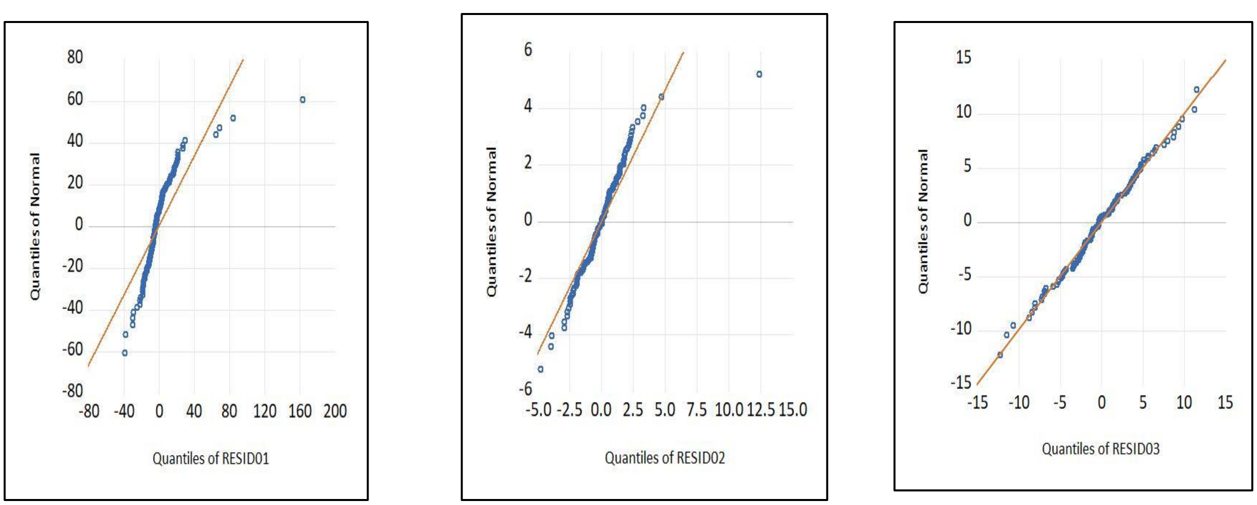


Figure 3: (c) Q-Q Plot for Sukkur Station

4.16 Forecast Error Variance Decomposition

FEV is used to investigate the dynamic interactions between the factors in the short term. Due to the monthly spacing of the data, the duration in this case is one month. Table 6(a, b, c) presents the processed degradation of FEV with projected horizons of 1, 6, and 12 months. The decomposition divides the estimated variance into numerous components, each of which may be explained by an original idea. Table 6 reveals that at Karachi station, there is an observable increase of over 18% in rainfall after 12 months due to temperature variation. Additionally, after 18 months, the variation in humidity leads to a more than 11% increase in rainfall.

Similarly, in Table 7 for Hyderabad station, the data indicates that variations in temperature and humidity result in rainfall increases of more than 12% and 10% after 12 and 18 months, respectively. Concurrently, Table 8 demonstrates that at Sukkur station, rainfall experiences an uptick after 12 months due to variations in both temperature and humidity, recording increases of more than 9% and 5%, respectively. The study concludes that both temperature and humidity positively influence rainfall, with maximum precipitation occurring when both factors are high and minimal rainfall observed when both are low.

Table 6: Forecast Error Variance Decomposition for VAR (8) Model at Karachi Station

FEV	Period (month)	Standard Error	Rainfall	Max. Temperature	Humidity
Rainfall	01	43.221	100.000	0.000	0.000
	06	44.839	94.420	2.258	3.322
	12	47.107	86.457	7.131	6.412
	18	48.159	84.667	8.081	7.252
Temperature	01	1.409	1.007	98.993	0.000
	06	2.110	11.616	64.418	23.965
	12	2.406	18.178	57.339	24.482
	18	2.826	15.902	57.991	26.107
Humidity	01	4.849	2.101	0.994	96.905
	06	5.494	4.833	11.753	83.414
	12	6.674	10.810	26.613	62.577
	18	7.364	11.452	26.638	61.909

Table No 7: Forecast Error Variance Decomposition for VAR (8) Model at Hyderabad Station

FEV	Period (month)	Standard Error	Rainfall	Max. Temperature	Humidity
Rainfall	01	25.946	100.000	0.000	0.000
	06	26.948	95.072	3.310	1.618
	12	28.541	85.137	10.858	4.005
	18	29.003	84.221	11.157	4.622
Temperature	01	1.801	4.411	95.589	0.000
	06	2.613	7.049	90.834	2.116

	12	2.824	12.560	83.771	3.669
	18	3.478	9.732	87.676	2.592
Humidity	01	4.969	0.033	0.019	99.948
	06	5.396	3.985	8.782	87.233
	12	6.130	9.026	16.141	74.834
	18	6.359	10.758	18.577	70.665

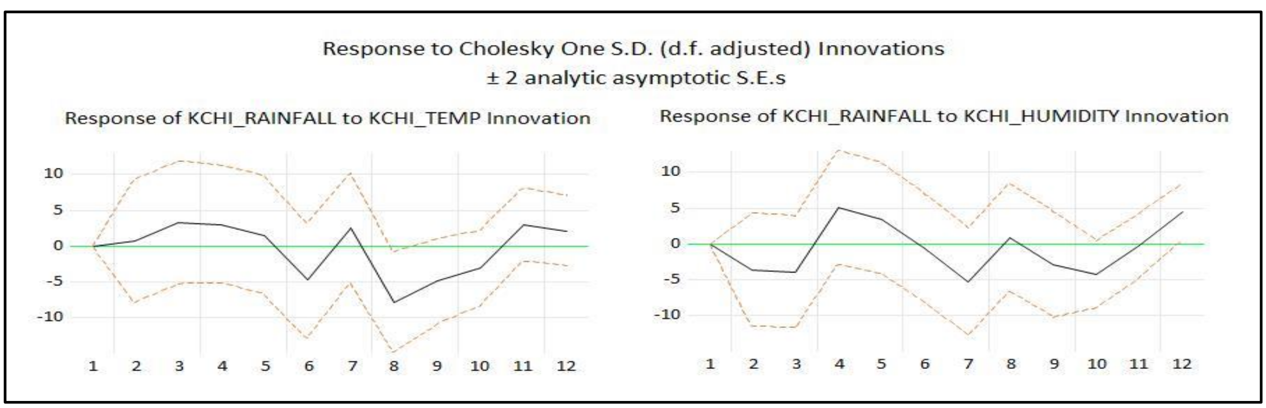
Table No 8: Forecast Error Variance Decomposition for VAR (7) Model at Sukkur Station

FEV	Period (month)	Standard Error	Rainfall	Max. Temperature	Humidity
Rainfall	01	25.121	100.000	0.000	0.000
	06	27.086	87.455	7.961	4.583
	12	27.881	83.352	11.219	5.429
	18	27.965	83.005	11.374	5.621
Temperature	01	2.152	6.620	93.379	0.000
	06	3.483	7.588	88.732	3.679
	12	3.812	9.808	83.758	6.432
	18	4.501	8.729	86.237	5.033
Humidity	01	5.066	5.839	9.296	84.864
	06	7.200	3.494	34.300	62.206
	12	7.545	5.613	36.703	57.684
	18	8.306	5.287	46.305	48.408

4.17 Impulse Response Function

In a VAR (p) model, impulse response functions (IRFs) are utilised to examine the impacts of shocks or impulses. In a VAR model, it illustrates the impact of a single unit or standard deviation shock to one endogenous variable on all other endogenous variables. Figure 4 depicts how rainfall changes over time in response to temperature and humidity increases.

Figure No 4: VAR (p) Model



4.18 Forecasting Accuracy

The results of the computation of the RMSE, MAE, and MAPE values for each weather station is shown in Table 7 (a, b, c), which is used to determine the accuracy of data processing. The results of the data processing generally show quite significant values; therefore, it can be said that the chosen VAR (p) models perform rather well in predicting the amount of rainfall in the research region. The graphical representation of rainfall forecasting is also given in figure 5 for all stations.

Table No 9: Forecasting Accuracy of Data at Karachi Station

	RMSE	MAE	MAPE	Theil
Rainfall	43.390	22.367	NA	0.635
Temperature	2.416	1.910	6.053	0.037
Humidity	6.549	5.306	7.926	0.045

Table No 10: Forecasting Accuracy of Data at Hyderabad Station

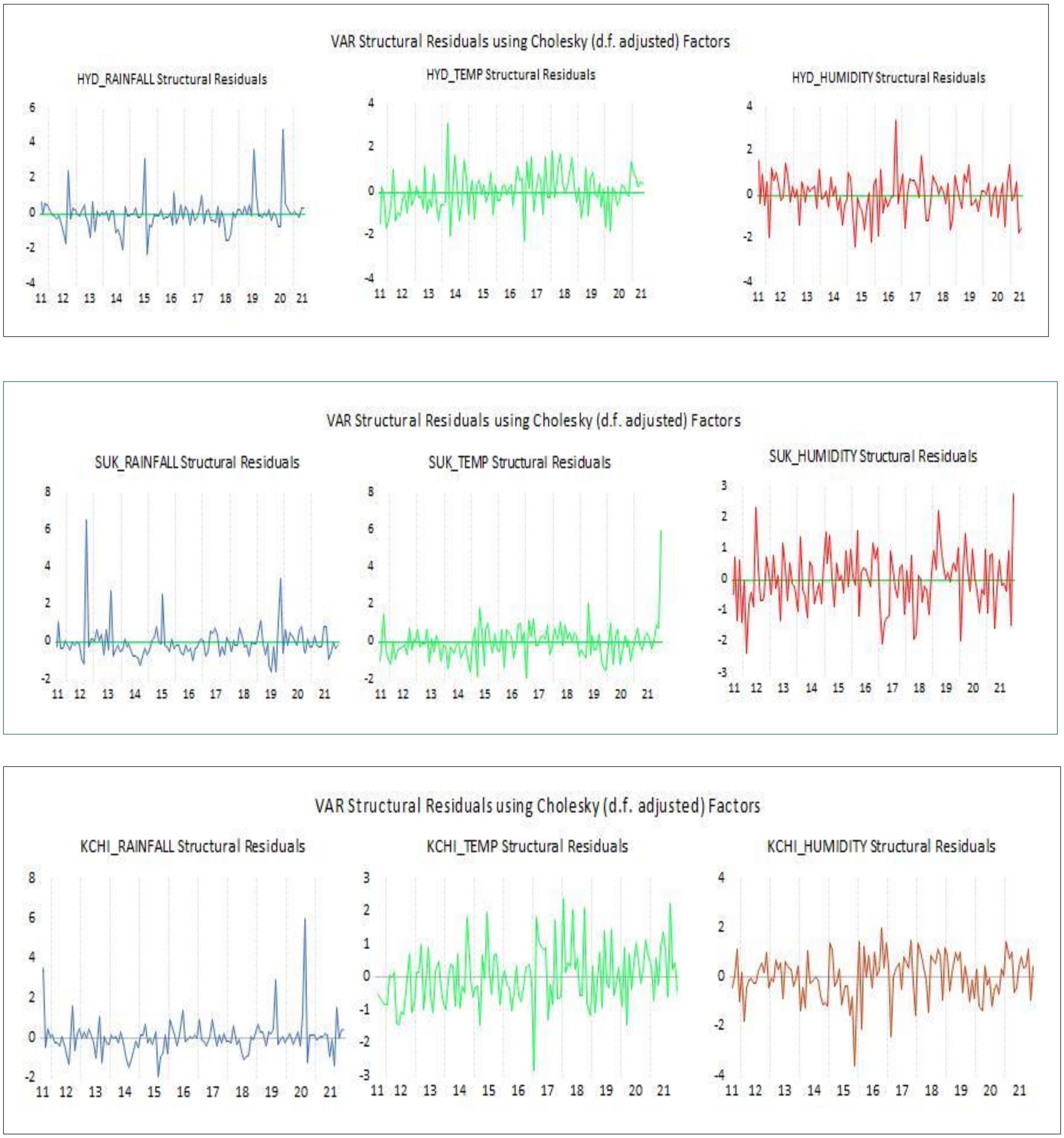
	RMSE	**MAE**	**MAPE**	**Theil**
Rainfall	25.786	13.097	NA	0.570
Temperature	2.874	2.243	7.170	0.042
Humidity	5.517	4.487	6.409	0.037

Table 11: Forecasting Accuracy of Data at Sukkur Station

	RMSE	MAE	MAPE	Theil
Rainfall	24.958	13.114	NA	0.643
Temperature	4.134	3.347	10.913	0.059

Humidity	7.370	5.565	8.688	0.052

Figure 5: Residuals Plots for Rainfall, Temperature and Humidity of VAR(p) Model



5. Conclusion

In recent times, the analysis of climate data has held significant importance. This study involved an analysis of rainfall, maximum temperature, and humidity from January 2011 to December 2021. In our study, the variables considered show stationarity at the level of bidirectional causation among themselves, suggesting a robust interconnection that could be effectively modelled using VAR analysis. The suitable order of lag length for the VAR model was determined utilizing AIC, SC, HQ, FPE, and LRT. The analysis revealed a lag order of 8 for Karachi and Hyderabad, whereas Sukkur stations had a lag order of 7.

Diagnostic checks on the VAR (p). According to the model, the residuals were almost normally distributed, stationary, and non-autocorrelated. Additionally, all fitted VAR models underwent thorough cross-validation. It is also deduced that elevated temperatures and humidity imply the presence of water vapour in the air, leading to an increase in atmospheric pressure and a heightened likelihood of precipitation. Conversely, high humidity signifies more moisture in the air, increasing the probability of cloud formation and rainfall if the temperature decreases. According to the study, both temperature and humidity exert a positive influence on rainfall, with maximum precipitation occurring when both factors are high and minimal rainfall observed when both are low.

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